

Antonio Navarra  
Valeria Simoncini

# A Guide to Empirical Orthogonal Functions for Climate Data Analysis

INPE - Bit  
CPTEC

26873



379154

551.58

N227g

2010

Navarra, Antonio

Guide to empirical orthogonal functions for climate  
data analysis

Springer



Dr. Antonio Navarra  
Ist. Nazionale di Geofisica e  
Vulcanologia  
Via Gobetti, 101  
40100 Bologna  
Italy  
navarra@ingv.it

Prof. Valeria Simoncini  
Università di Bologna  
Dip. to Matematica  
Piazza di Porta San Donato, 5  
40126 Bologna  
Italy

## Contents

<b>1</b>	<b>Introduction</b>	1
<b>2</b>	<b>Elements of Linear Algebra</b>	5
2.1	Introduction	5
2.2	Elementary Vectors	5
2.3	Scalar Product	6
2.4	Linear Independence and Basis	10
2.5	Matrices	12
2.6	Rank, Singularity and Inverses	16
2.7	Decomposition of Matrices: Eigenvalues and Eigenvectors	17
2.8	The Singular Value Decomposition	19
2.9	Functions of Matrices	21
<b>3</b>	<b>Basic Statistical Concepts</b>	25
3.1	Introduction	25
3.2	Climate Datasets	25
3.3	The Sample and the Population	26
3.4	Estimating the Mean State and Variance	27
3.5	Associations Between Time Series	29
3.6	Hypothesis Testing	32
3.7	Missing Data	36
<b>4</b>	<b>Empirical Orthogonal Functions</b>	39
4.1	Introduction	39
4.2	Empirical Orthogonal Functions	42
4.3	Computing the EOFs	43
4.3.1	EOF and Variance Explained	44
4.4	Sensitivity of EOF Calculation	49
4.4.1	Normalizing the Data	51
4.4.2	Domain of Definition of the EOF	55
4.4.3	Statistical Reliability	58
4.5	Reconstruction of the Data	58

Additional material to this book can be downloaded from <http://extra.springer.com>.

ISBN 978-90-481-3701-5 e-ISBN 978-90-481-3702-2  
DOI 10.1007/978-90-481-3702-2  
Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2010920466

© Springer Science+Business Media B.V. 2010

No part of this work may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission from the Publisher, with the exception of any material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work.

Cover design: Boekhorst Design b.v.

Printed on acid-free paper

Springer is part of Springer Science+Business Media ([www.springer.com](http://www.springer.com))

4.5.1	The Singular Value Distribution and Noise	59
4.5.2	Stopping Criterion	62
4.6	A Note on the Interpretation of EOF	64
<b>5</b>	<b>Generalizations: Rotated, Complex, Extended and Combined EOF</b>	<b>69</b>
5.1	Introduction	69
5.2	Rotated EOF	70
5.3	Complex EOF	79
5.4	Extended EOF	87
5.5	Many Field Problems: Combined EOF	90
<b>6</b>	<b>Cross-Covariance and the Singular Value Decomposition</b>	<b>97</b>
6.1	The Cross-Covariance	97
6.2	Cross-Covariance Analysis Using the SVD	99
<b>7</b>	<b>The Canonical Correlation Analysis</b>	<b>107</b>
7.1	The Classical Canonical Correlation Analysis	107
7.2	The Modes	109
7.3	The Barnett-Preisendorfer Canonical Correlation Analysis	114
<b>8</b>	<b>Multiple Linear Regression Methods</b>	<b>123</b>
8.1	Introduction	123
8.1.1	A Slight Digression	125
8.2	A Practical PRO Method	126
8.2.1	A Different Scaling	127
8.2.2	The Relation Between the PRO Method and Other Methods	128
8.3	The Forced Manifold	129
8.3.1	Significance Analysis	136
8.4	The Coupled Manifold	141
	<b>References</b>	<b>147</b>
	<b>Index</b>	<b>149</b>

## Chapter 1 Introduction

Climatology and meteorology has been basically a descriptive science without the means to perform quantitative experiments under controlled conditions. In fact, until the second half of the twentieth century, the border between climatology and geography was often blurred and the two disciplines were confused one with the other. The situation changed when the solution of the evolution equations for the climate system became possible using numerical methods. The development of numerical models allowed the application of standard scientific verification machinery for testing hypotheses, but crucial to the success of the strategy is that the model must be a good representation of the real climate system of the Earth. Assessing the quality of models regarding their capability to reproduce the climate became a cornerstone in the scientific progress of climatology. Tighter and tighter standards were required for the model simulations in comparison with the real characteristics of climate. Models were required to reproduce not only the mean properties of climate, but also its variability. In the last decades of the XX century the amount of data available was becoming very large and strong evidence of remote spatial relations between climate variability in geographically diverse regions were emerging. Quantitative techniques were developed to explore the climate variability and its relations among different geographical locations. Methods were borrowed from descriptive statistics, where they were developed to analyze variance of related observations-variable pairs, or to identify unknown relations among variables.

These methods were introduced to meteorology in the mid-1960, but they became increasingly popular in the early 1980s where their capability to identify dynamically significant modes in the climate variability was demonstrated. Since then they have been further developed and many variants and extensions have been proposed and applied. Very often these developments were taking place separately from the formal development in the mainstream statistics and reflected ad hoc solution to the particular vies that climatology was using.

There are excellent books treating these methods in a formal and rigorous way (von Storch and Zwiers 1999; Wilks 2005; Jolliffe 2002) and we refer the reader to these excellent texts for proofs and a more formal treatment. We take in this booklet a different approach, trying to introduce the reader to a practical application of the methods and to the kind of real problems that can be encountered in a practical application. We are including in the book data sets from real simulations